Slides are available on my website.
http://lukaseigentler.github.io

Can we predict wavelength changes of banded vegetation patterns?

BAMC 2026

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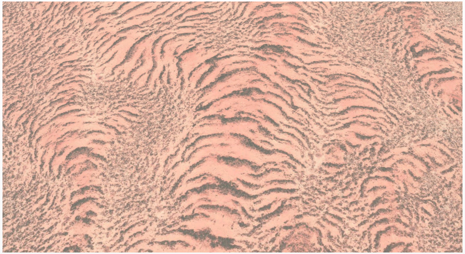
joint work with Mattia Sensi (University of Trento, Italy)

Stripe patterns

Banded vegetation patterns are classic examples of self-organisation principles in ecology.

Vegetation stripes in Ethiopia.



Vegetation stripes in Niger.

- Parallel to topographic contours.
- Caused by a scale-dependent feedback loop comprising long-range competition for water and short-range facilitation due to increased water infiltration.

Klausmeier model for vegetation patterns

One of the most basic phenomenological models for vegetation patterns is the extended Klausmeier reaction-advection-diffusion model.[1]

$$\frac{\partial u}{\partial t} = \overbrace{u^2 w}^{\text{plant growth}} - \overbrace{Bu}^{\text{plant loss}} + \overbrace{\frac{\partial^2 u}{\partial x^2}}^{\text{plant dispersal}},$$

$$\frac{\partial w}{\partial t} = \underbrace{A}_{\text{rainfall}} - \underbrace{w}_{\text{evaporation}} - \underbrace{u^2 w}_{\substack{\text{water uptake}\\ \text{by plants}}} + \underbrace{\nu \frac{\partial w}{\partial x}}_{\substack{\text{water flow}\\ \text{downhill}}} + \underbrace{d \frac{\partial^2 w}{\partial x^2}}_{\substack{\text{water}\\ \text{diffusion}}}.$$

[1] Klausmeier, C. A.: *Science* 284.5421 (1999).

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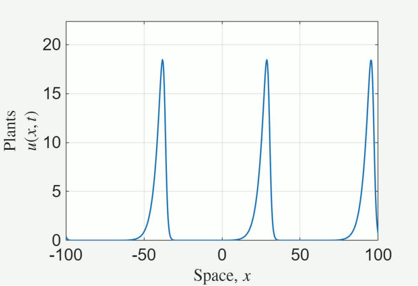
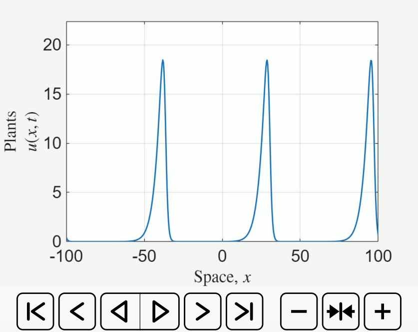
Periodic travelling waves



alternative video link.

- Model represents vegetation patterns as periodic travelling waves (PTWs).

Periodic travelling waves



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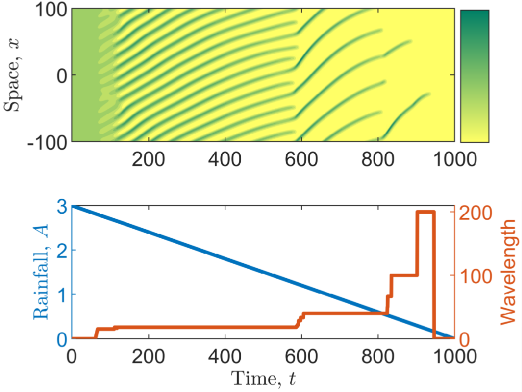
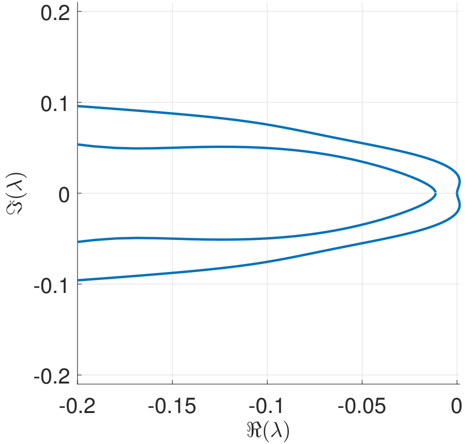


- Model represents vegetation patterns as periodic travelling waves (PTWs).
- Along rainfall gradient, transition from uniform vegetation to desert occurs via several pattern transitions.

Wavelength changes

- State-of-the-art: predict wavelength changes through PTW stability properties.
- PTW linear stability is determined by their essential spectra.
- Calculated using numerical continuation.[a]

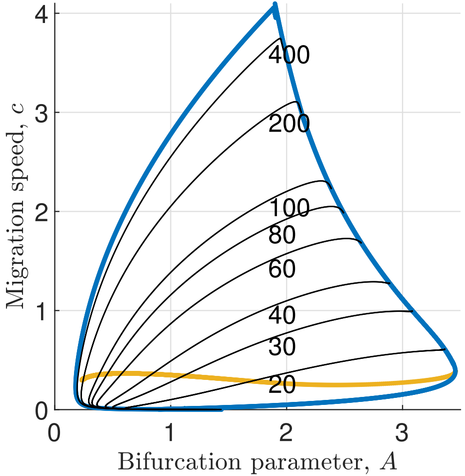
[a] Rademacher, J. D., Sandstede, B. and Scheel, A.: *Physica D* 229.2 (2007).



Wavelength changes

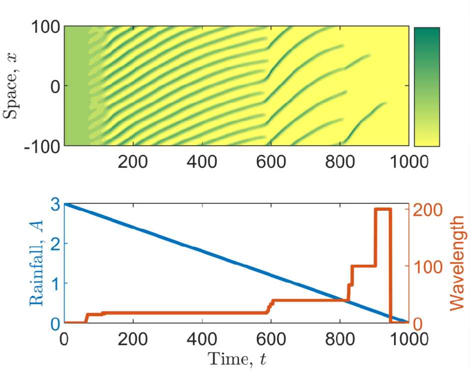
- State-of-the-art: predict wavelength changes through PTW stability properties.
- PTW linear stability is determined by their essential spectra.
- Calculated using numerical continuation.[a]
- Wavelengths changes are typically predicted through the *Busse balloon*: parameter space of stable PTWs.

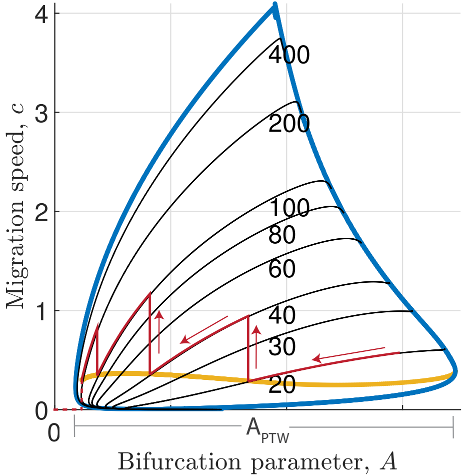
[a] Rademacher, J. D., Sandstede, B. and Scheel, A.: *Physica D* 229.2 (2007).



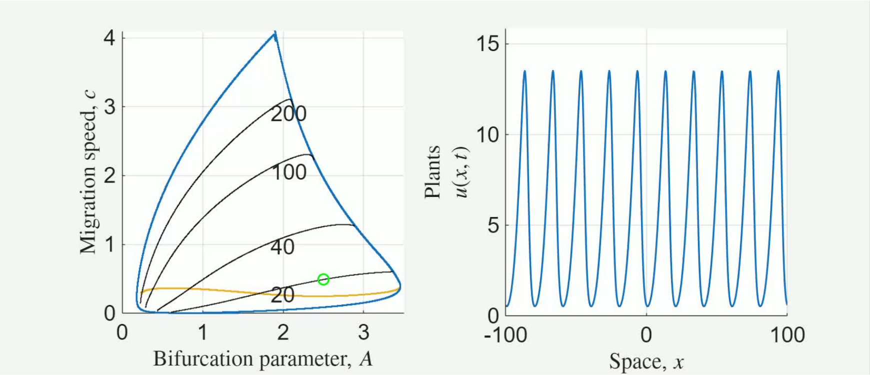
Wavelength changes

- Wavelengths are preserved, provided they remain stable.
- Upon destabilisation a wavelength change occurs.





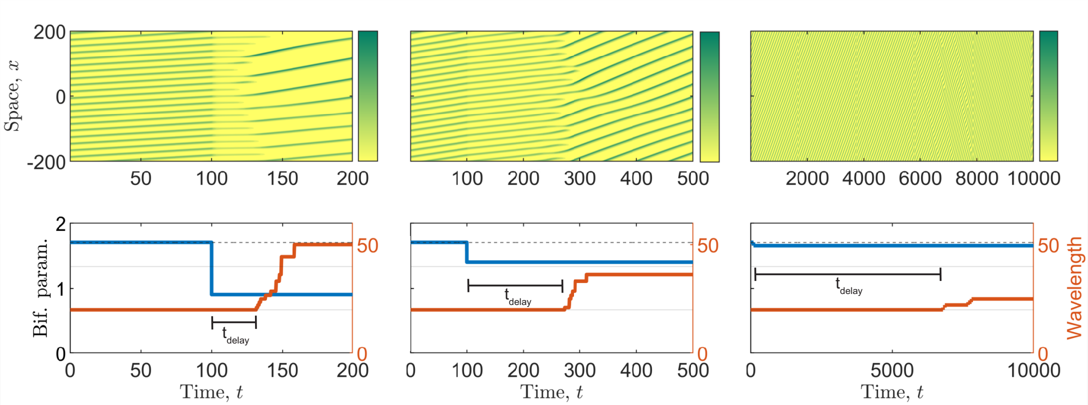
Wavelength changes



alternative video link.

- Significant delays between crossing a stability boundary and observing wavelength changes occur.

Delays to wavelength changes



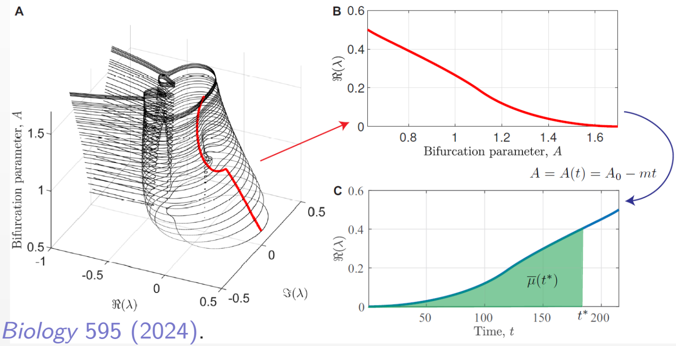
- Significant delays between crossing a stability boundary and observing wavelength changes occur.
- Order of magnitude differences in delay depending on parameter values.

Predicting delays

Can predict the order of magnitude of the delay through the accumulated maximal instability[2]

$$\overline{\mu}(A(t)) = \int_{t_{\text{stab}}}^{t} \mu(\tau)d\tau, \quad t \geq t_{\text{stab}}.$$

t_{stab} is the time of the last crossing of the stability boundary.
$\mu(t)$ is the max real part of the spectrum at time t.



[2] EL and Sensi, M.: *Journal of Theoretical Biology* 595 (2024).

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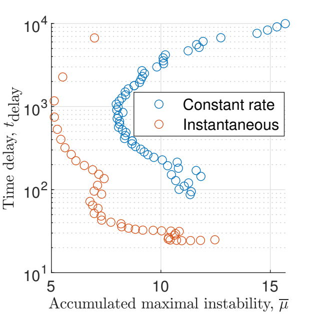
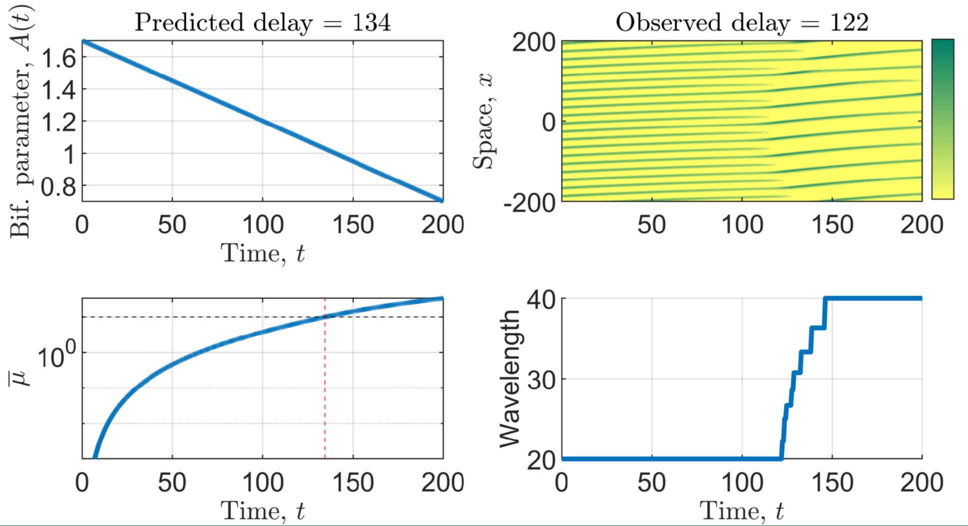


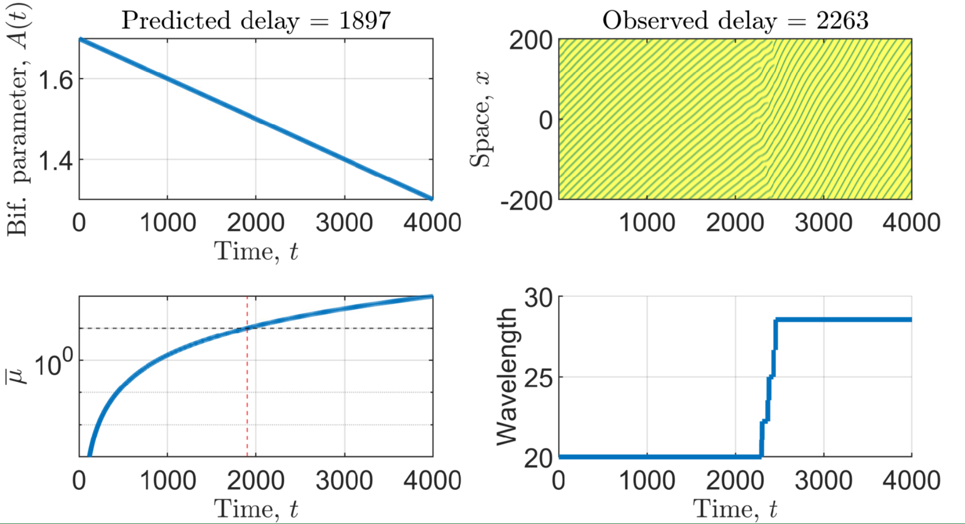
Wavelength change occurs when $\overline{\mu} \approx 10$

[2] EL and Sensi, M.: *Journal of Theoretical Biology* 595 (2024).

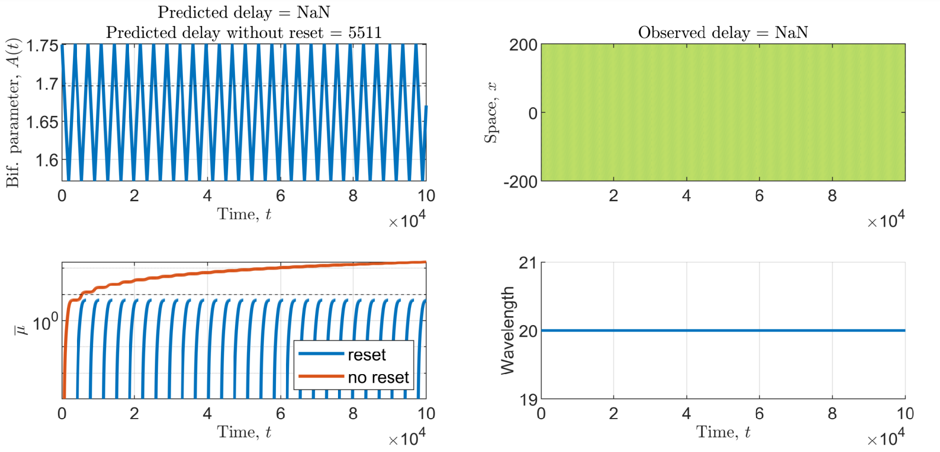
Delay prediction in practice



Delay prediction in practice

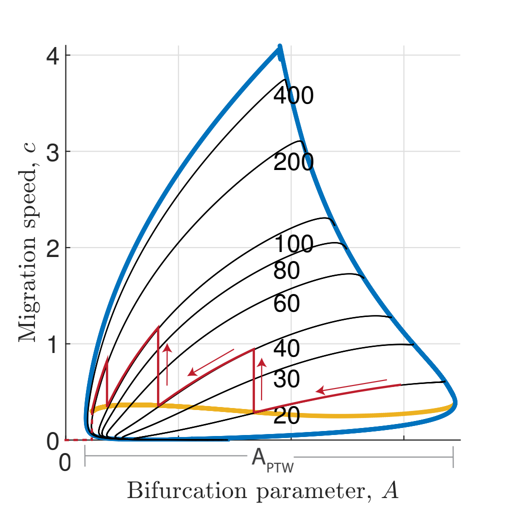


Delay prediction reset in stable regions



Conclusions

- Wavelength changes that occur after crossing a stability boundary are subject to a delay.



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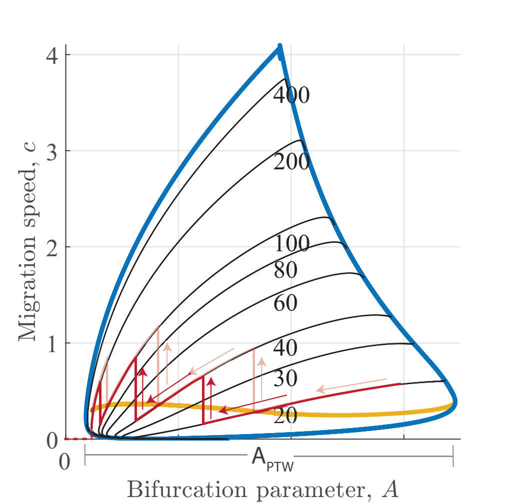
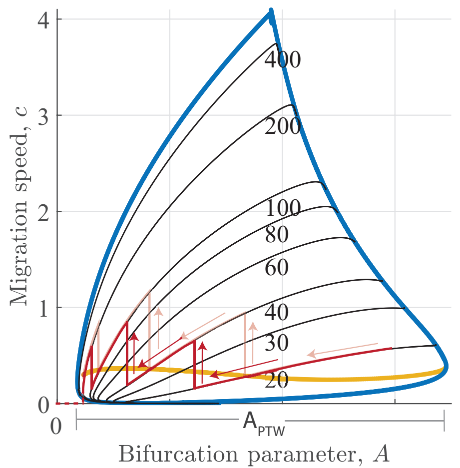


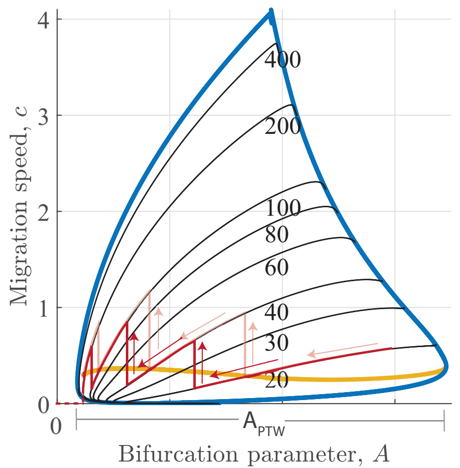
Conclusions

- Wavelength changes that occur after crossing a stability boundary are subject to a delay.
- Order of magnitude of the delay can be predicted by tracking the maximum real part of the spectrum of the destabilised pattern over time.



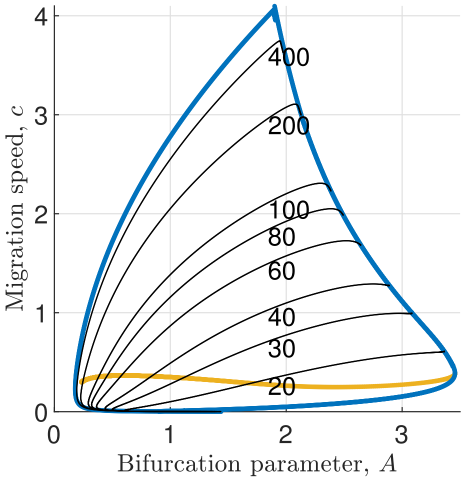
Conclusions

- Wavelength changes that occur after crossing a stability boundary are subject to a delay.
- Order of magnitude of the delay can be predicted by tracking the maximum real part of the spectrum of the destabilised pattern over time.
- Open question: What new wavelength is chosen?



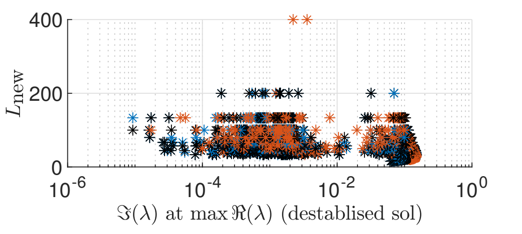
Wavelength changes

- Open question: What new wavelength is chosen?
- For fixed PDE parameters, there is multistability of different periodic travelling waves.



Linear analysis insufficient

- Created a large dataset of wavelength changes through simulations.
- Compared wavelength change dynamics with features of essential spectra.
- Suggests that linear analysis is insufficient to characterise wavelength changes.[a]

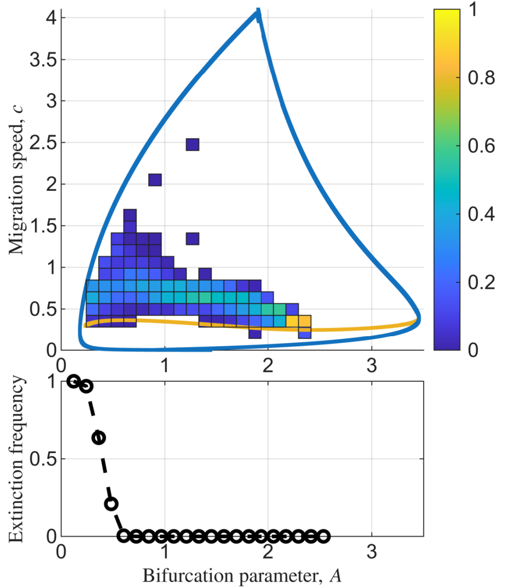


[a] EL and Sensi, M.: *Bull. Math. Biol.* 88.22 (2026).

Some selection data

- Large areas of Busse balloon remain unselected.
- Extinction does not necessarily occur at the edge of the Busse balloon.[a]

[a] EL and Sensi, M.: *Bull. Math. Biol.* 88.22 (2026).



What next?

- What (new) methods do we need to understand periodic travelling wave wavelength selection?
- Similar trends observed for mussel model $\Rightarrow$ is it possible to derive principles applicable to a wider class of models?
- Do we have empirical evidence of wavelength changes in dryland vegetation patterns?

References

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[1] Eigentler, L. and Sensi, M.: 'Wavelength selection for periodic travelling waves: an unsolved problem'. *Bull. Math. Biol.* 88.22 (2026).

[2] Eigentler, L. and Sensi, M.: 'Delayed loss of stability of periodic travelling waves: insights from the analysis of essential spectra'. *Journal of Theoretical Biology* 595 (2024), p. 111945.